Math 2D Quiz 6 Morning - March 3, 2016

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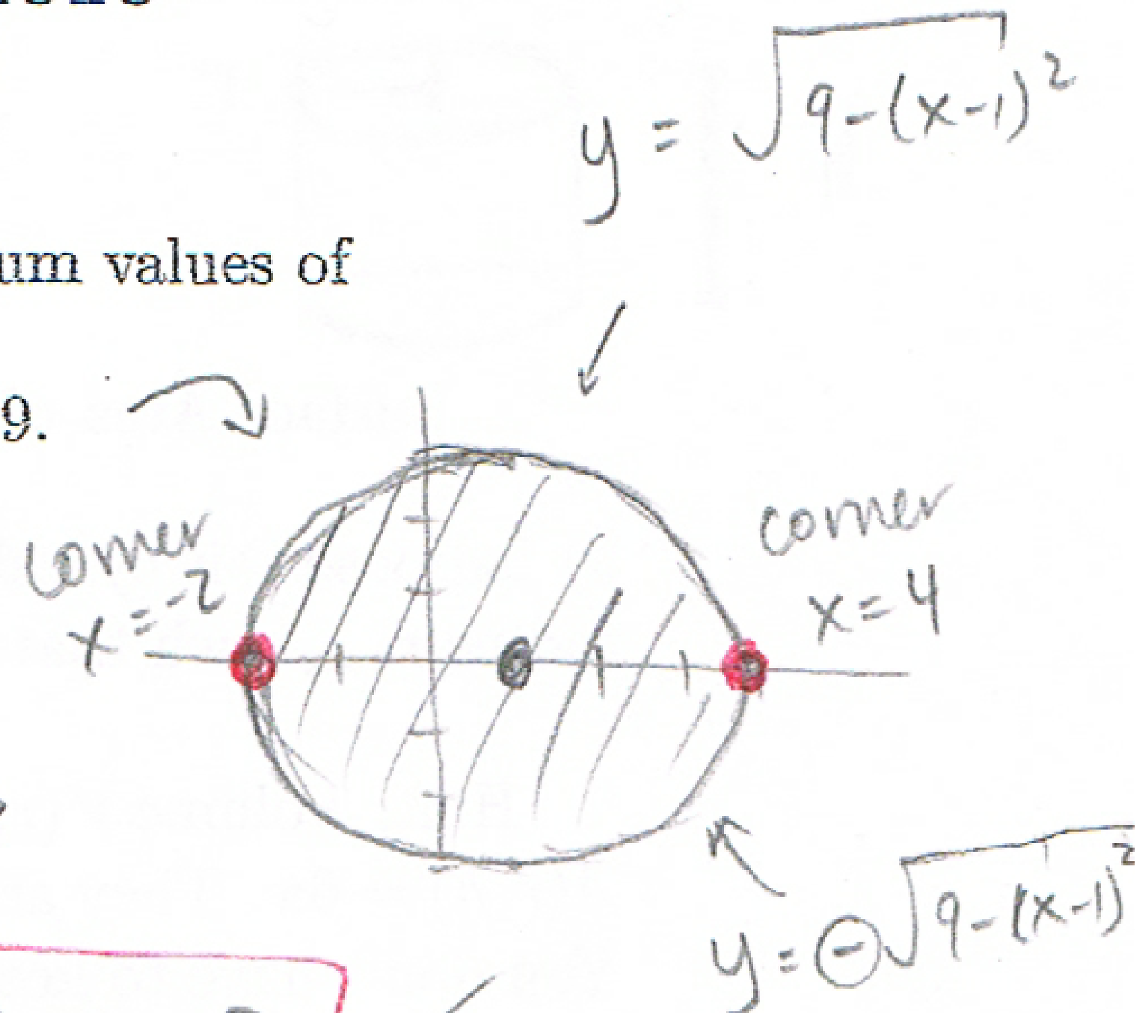
Show all of your work. *There is a question on the back side.

1. Without Lagrange Multipliers, find the absolute maximum and minimum values of

$$f(x,y) = x^2 + 2y^2 \quad \text{on the domain} \quad (x-1)^2 + y^2 \le 9.$$

*Be sure that you go through all of the necessary steps.



1) Find $\nabla f = 0$: $\nabla f(x,y) = \langle 2x, 4y \rangle$

This is zero at $x = y = 0$, $\boxed{f(0,0) = 0}$ ✓ +1

2) Check Boundary: We see $(x-1)^2 + y^2 = 9$ is the bdy,

so $y = \pm\sqrt{9-(x-1)^2}$.

Thus, we get • $g_+(x) = f(x, \sqrt{9-(x-1)^2}) = x^2 + 2(9-(x-1)^2)$.

(same function!) • $g_-(x) = f(x, \ominus\sqrt{9-(x-1)^2}) = x^2 + 2(9-(x-1)^2)$.

(Local) Maximizing, $g'_\pm(x) = 2x - 4(x-1) = 2x - 4x + 4 = 4 - 2x$.

This is zero at $x = 2$ $(y = \pm\sqrt{8})$, where $\boxed{g_\pm(2) = f(2, \pm\sqrt{8}) = 4 + 16 = \boxed{20}}$ +2

3) Last, check "corners" of domain ⇝ Here, its $x = -2, 4$ (where $y = 0$),

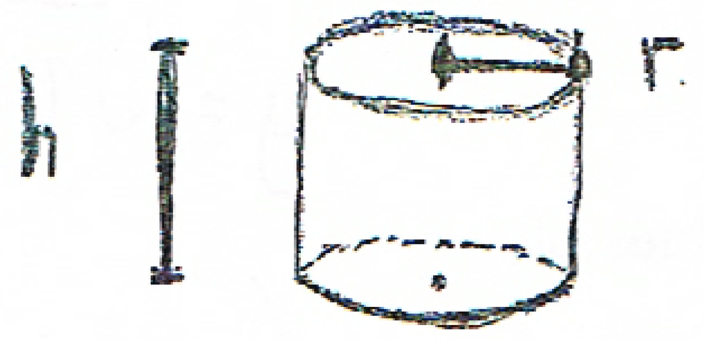
$g_\pm(-2) = 4 + 2(0) = 4$; $g_\pm(4) = 16 + 2(0) = 16$

+1 +1

We see:

$$\boxed{\begin{array}{l} \text{Absolute Min, } f(0,0) = 0 \\ \text{Absolute Max, } f(2, \pm\sqrt{8}) = 20 \end{array}}$$

2. a) Fill in the blank. Consider a cylinder of height h and radius r. From Tuesday's Discussion,



Volume of a Cylinder : $V(r,h) =$ $\pi r^2 h$.

Surface Area of a Cylinder, including the caps : $A(r,h) =$ $2\pi r^2 + 2\pi r h$. $= 6\pi \text{ cm}^2$.

b) Suppose this cylinder is constrained to have a surface area of 6π cm^2. Find the height $h_{\max}$ and radius $r_{\max}$ such that the cylinder has maximal volume, using Lagrange Multipliers.

Hint: Volume $V(r,h)$ is the function we are maximizing. The constraint is that the surface area $A(r,h) = 6\pi$. They are functions of r and h, so for example, $\nabla V(r,h) = <\frac{\partial V}{\partial r}, \frac{\partial V}{\partial h}> = <V_r, V_h>$. You don't have to keep track of the units, but it would be nice :)

$$\begin{cases} \nabla V = \lambda \nabla A \\ A = 6\pi \text{ cm}^2 \end{cases} \rightsquigarrow \begin{cases} \frac{\partial}{\partial r}: & 2\pi r h = \lambda(4\pi r + 2\pi h) & \text{(i)} \\ \frac{\partial}{\partial h}: & \pi r^2 = 2\pi \lambda r & \text{(ii)} \\ A = 6\pi: & 2\pi r^2 + 2\pi r h = 6\pi & \text{(iii)} \end{cases}$$

Pick on (ii) If in (ii) we had $0 = 0 \implies r = 0$ on LHS (V=0, min.)

Thus, we can now assume it's not $0=0$, so $\cancel{\pi} r^{\cancel{2}} = 2\cancel{\pi}\cancel{r}\lambda$, where r and λ are not 0.

This implies $\boxed{r = 2\lambda}$. +1

Plug this into (i), $4\pi \cancel{\lambda} h = \cancel{\lambda}(8\pi\lambda + 2\pi h)$,

$\iff 4\pi h = 2\pi h + 8\pi\lambda$; $2\pi h = 8\pi\lambda$

In other words, $\boxed{h = 4\lambda}$ +1

• $r = 2\lambda$ so $\boxed{h = 2r}$, so going to (iii),

$2\pi r^2 + 4\pi r^2 = 6\pi$; $r^2 = 1$ (cm^2), $\boxed{\begin{matrix} r_{\max} = 1 \text{ cm} \\ h_{\max} = 2 \text{ cm} \end{matrix}}$ +1

since $h = 2r$,